
Deep Energy-Based Generative Modeling and Learning

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Ph. D. Defense

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Self-introduction

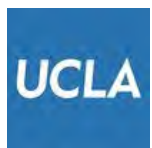
徐亦飞 Yifei Xu



- 2003 – 2013 : Shanghai Experimental School (K-12)
 - Primary, Mid, High School (K-12)



- 2013 – 2017 : Shanghai Jiao Tong University
 - Bachelor of Engineering --- **Computer Science**
 - Zhiyuan College ACM Honored Class
 - Advisor: Liqing Zhang



- 2017 – Now : University of California, Los Angeles
 - Doctor of Philosophy --- **Statistics**
 - Advisor: Ying Nian Wu



Knowledge Representation: Concepts and Models

Concept $\longleftrightarrow$ Set $\longleftrightarrow$ Model



A concept, e.g., chair
(a set of 3D chair object)

Energy-based Model

Concept $\longleftrightarrow$ Set $\longleftrightarrow$ Model

$$p_{\theta}(X) = \frac{1}{Z(\theta)} \exp f_{\theta}(X) p_0(X)$$



An object



A concept, e.g., chair
(a set of 3D chair object)

Energy-based Model

Directly model the probability:

$$\log p_{\theta}(X) \propto f_{\theta}(X)$$

$$f_{\theta}(X): \mathbf{R}^D \rightarrow \mathbf{R}$$

Any differentiable function
e.g. weight sum of a heuristic rule,
Gabor filter on image, or neural network.

$$p_{\theta}(X) = \frac{1}{Z(\theta)} \exp f_{\theta}(X) p_0(X)$$

$$Z(\theta) = \int \exp f_{\theta}(X) dX$$

The normalization constant to ensure overall probability sum up to 1.

$$p_0(X) \sim N(0, I_D)$$

The white noise prior distribution.

Discriminative, generative and descriptive



Discriminative Task

$$p_{\theta}(C|X)$$



Generative Task

$$p_{\theta}(X|z)$$

Discriminative, generative and descriptive



Descriptive Model
 $p_{\theta}(X)$



Discriminative Model
 $p_{\theta}(C|X)$



Generative Model
 $p_{\theta}(X|z)$

Discriminative, generative and descriptive



Descriptive Model
 $p_{\theta}(X)$



Discrimination
$$p_{\theta}(k|X) = \frac{\exp f_{\theta_k}(X)}{\sum_{l=1}^K \exp f_{\theta_l}(X)}$$



Generation
 $X \sim p_{\theta}(X)$

Advantage of EBM

Simplicity

Directly model the probability.

Stability

No need assisting network to ensure balance.

Flexibility

Any bottom-up function can act as energy.

Adaptivity

Avoid mode collapse and avoiding spurious modes from out-of-distribution samples.

Compositionality

Models to be combined through product of experts or other hierarchical techniques.

Everything to generate



Learning

Representing

Controlling

Learning

How to model a set?

Generation? Reconstruction?

Semi-supervised representation learning?

Representing

How to represent a 3D shape?

Voxel? Point Cloud? Mesh?

A function itself can be a form of data representation?

How EBM works with VAE?

Controlling

What is inverse optimal control?

How to control a vehicle driving on the road?

How to control if we do not even know what good is?

How to do control efficiently and accurately?

Learning

**Generative PointNet: Energy-Based
Learning on Unordered Point Sets**

Representing

**Energy-based Implicit Function
for 3D Shape Representation**

Controlling

**Energy-based Continuous
Inverse Optimal Control**

0. Fundamental

- Train an EBM using MLE
- Sample-based Approximation

Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

Representing

Energy-based Implicit Function for 3D shape representation

Controlling

Energy-based Continuous Inverse Optimal Control

Energy-based Model --- Training

- Maximum Likelihood Estimation:

$$l(\theta) = E_{q_{data}}[\log p_{\theta}(X)] \approx \frac{1}{n} \sum_{i=1}^n \log p_{\theta}(X_i)$$

- Train model by gradient descent:

$$\frac{\partial}{\partial \theta} l(\theta) = E_{q_{data}} \left[\frac{\partial}{\partial \theta} f_{\theta}(X) \right] - E_{p_{\theta}} \left[\frac{\partial}{\partial \theta} f_{\theta}(X) \right] \approx \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \theta} f_{\theta}(X_i) - \frac{1}{n} \sum_{i=1}^n \frac{\partial}{\partial \theta} f_{\theta}(\tilde{X}_i)$$



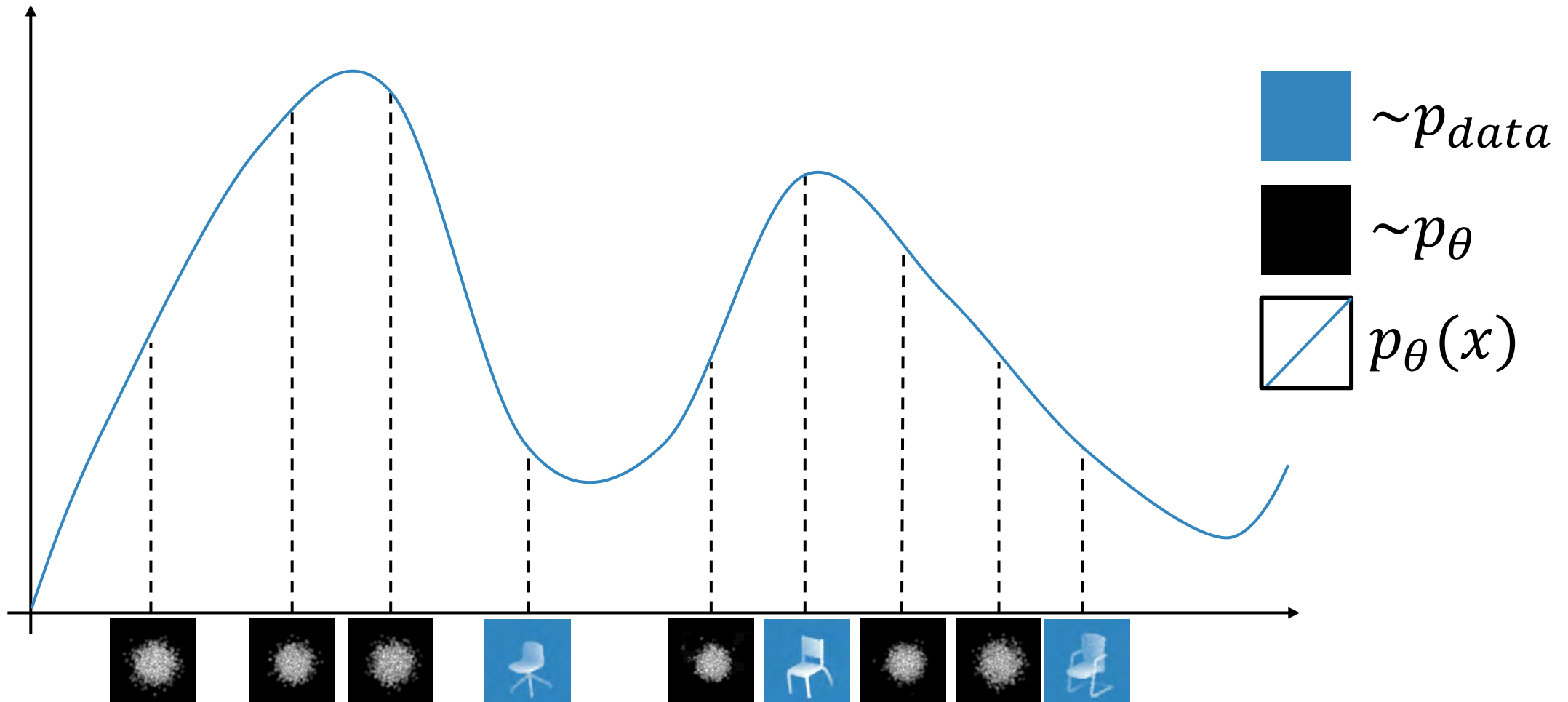
$\sim p_{data}$



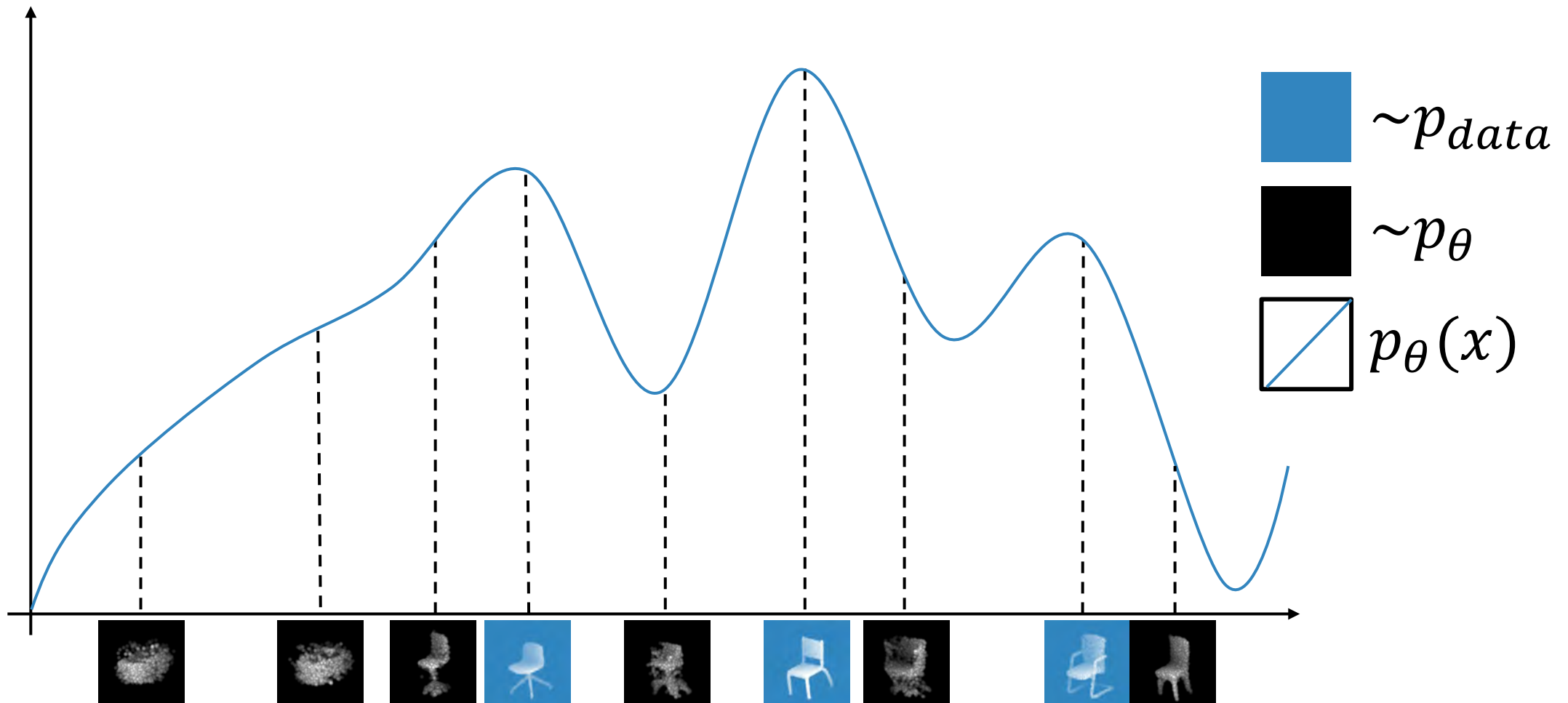
$\sim p_{\theta}$

Use MCMC sampling

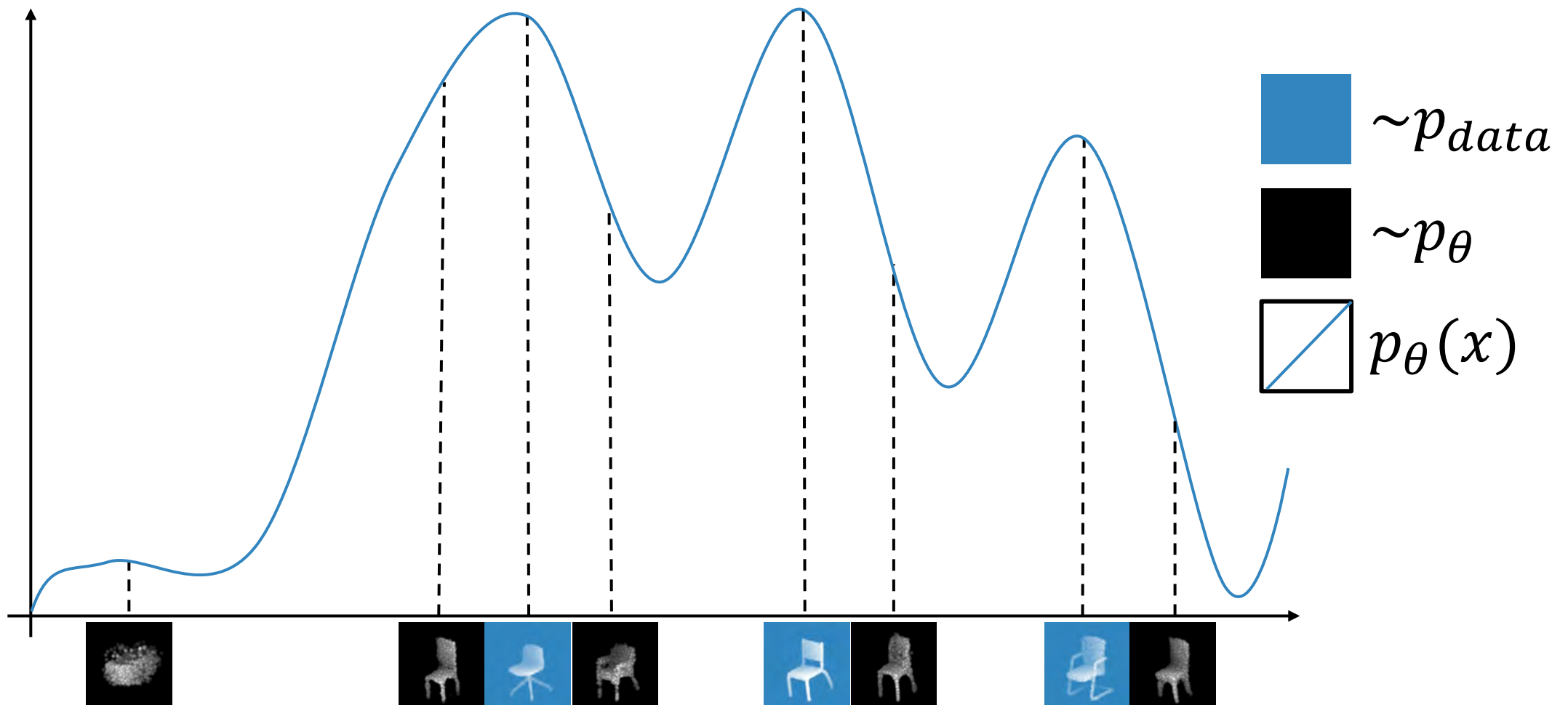
Energy-Based Model --- Training



Energy-Based Model --- Training



Energy-Based Model --- Training



1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

2. Representing

Energy-based Implicit Function for 3D shape representation

3. Controlling

Energy-based Continuous Inverse Optimal Control

Current challenges?

Why EBM helps?

How to model and sample?

One more thing...

1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta} \quad \text{on}$$

Energy-Based Model

2. Representing

Energy-based Implicit Function for 3D shape representation

3. Controlling

Energy-based Continuous Inverse Optimal Control



Point Clouds

Point Cloud

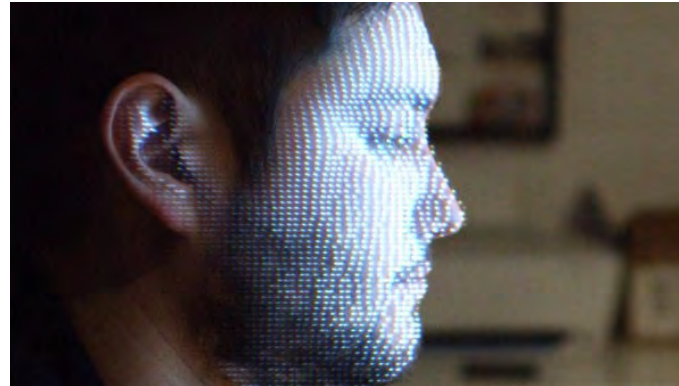
Why Special?

Unordered set

$$\{x, y, z\} = \{y, z, x\}$$



Scene Reconstruction



Face ID



Lidar in AV

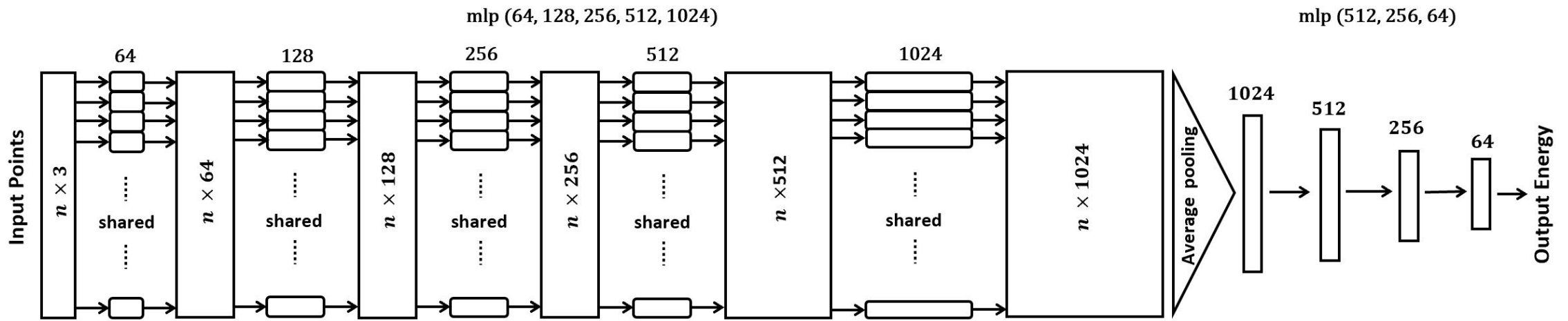
Input-permutation-invariant Score Function

- **Energy-Based Model on point cloud:**

$$p_{\theta}(X) = \frac{1}{Z(\theta)} \exp f_{\theta}(X) p_0(X)$$

$Z(\theta)$: Normalizing Constant; $p_0(X)$: prior distribution

$f_{\theta}(X)$ is parameterized by a bottom-up input-permutation-invariant neural network.



$$f_{\theta}(\{x_1, \dots, x_M\}) = g(\{h(x_1), \dots, h(x_m)\})$$

Energy-based Model --- Sampling

- Langevin Dynamics MCMC sampling:

$$X_0 = N(0, \sigma^2)$$
$$X_{\tau+1} = X_{\tau} + \underbrace{\frac{\delta^2}{2} \frac{\partial}{\partial X} f_{\theta}(X_{\tau})}_{\text{Transformation}} + \underbrace{\delta U_{\tau}}_{\text{Noise}}$$

where $U_{\tau} \sim N(0, 1)$;

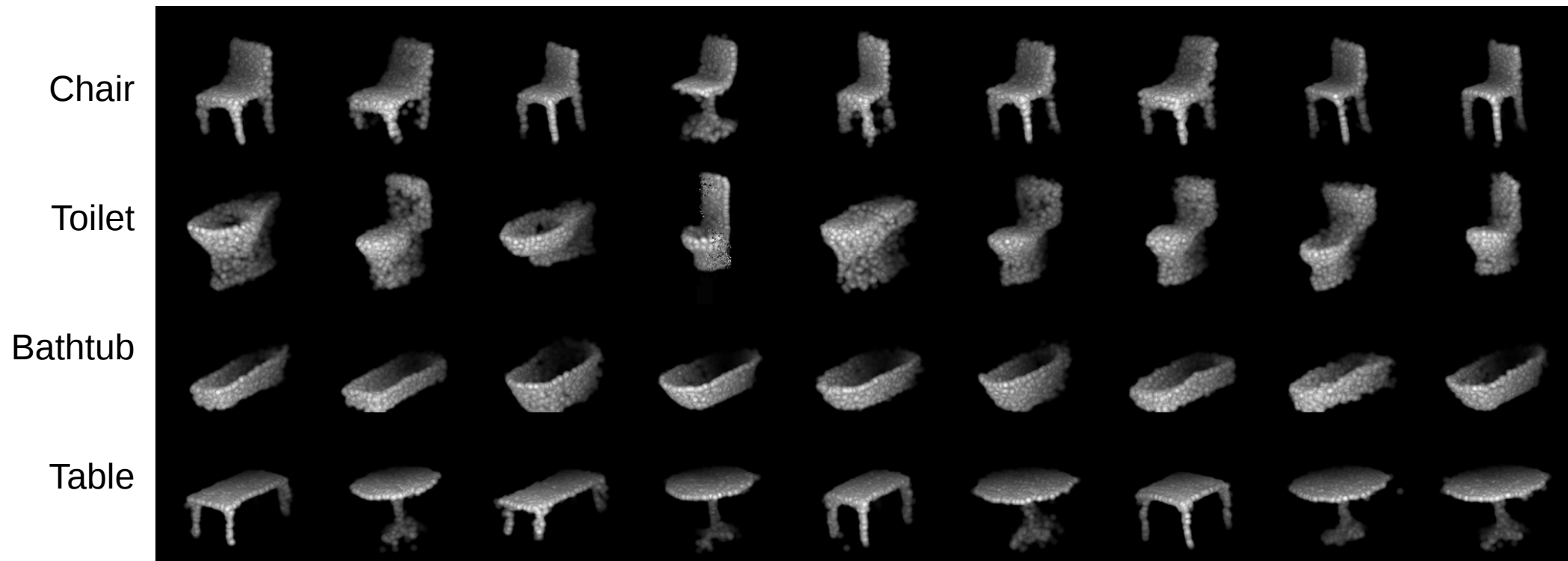
- (K -step) Short-run MCMC generator:

Short-run MCMC procedure $\xrightarrow{\text{regard as}}$ K -layer generator model

$$\tilde{X} = M_{\theta}(Z, \xi), \quad Z \sim p_0(Z)$$

Generation Results

- We synthesize 3D point clouds by short-run MCMC sampling from the learned model.

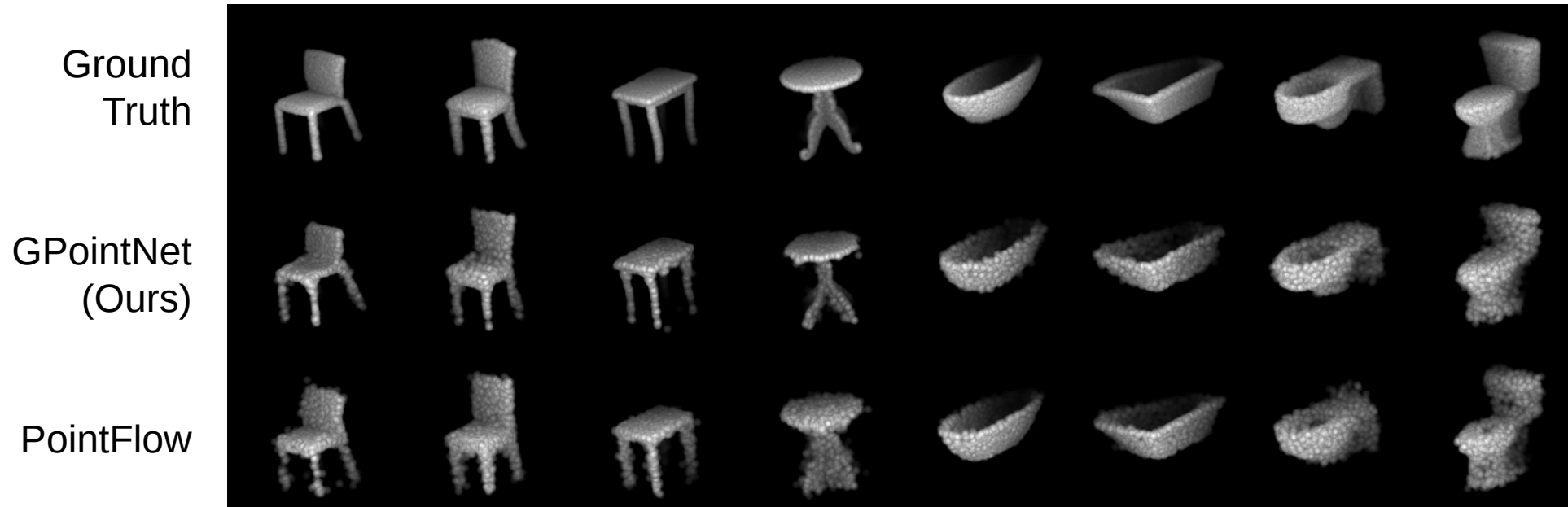


Lowest quantitatively score in **8 / 10** category

Reconstruction Results

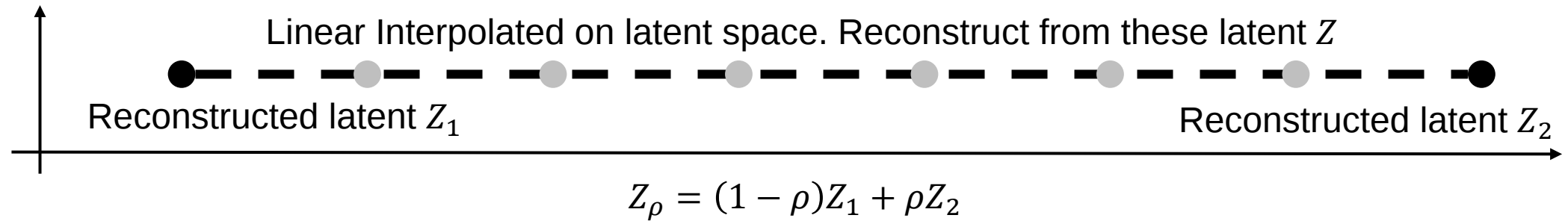
- Short-run MCMC procedure $\xrightarrow{\text{regard as}}$ K -layer generator model $M_{\theta}(Z, \xi)$

$$Z = \arg \min_Z L(Z) = \|X - M_{\theta}(Z)\|^2$$

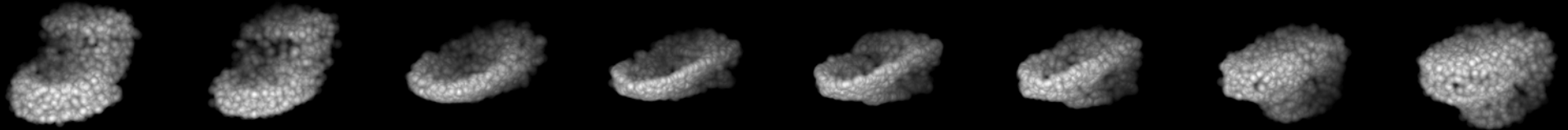


Lowest reconstruction loss in **ALL** category

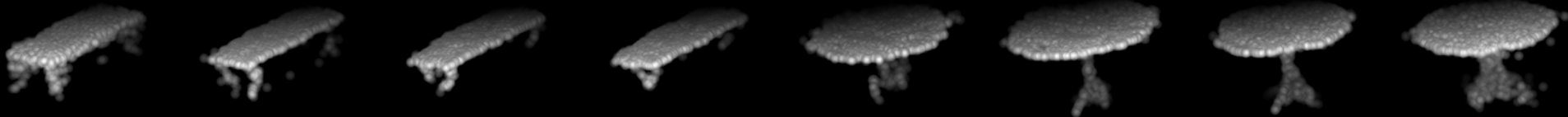
Interpolation Results



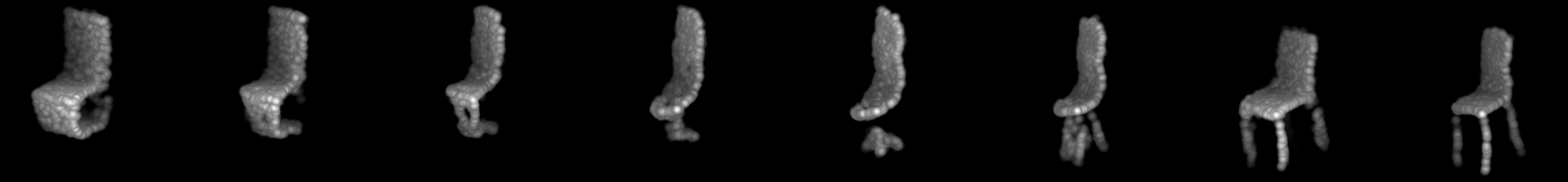
Toilet



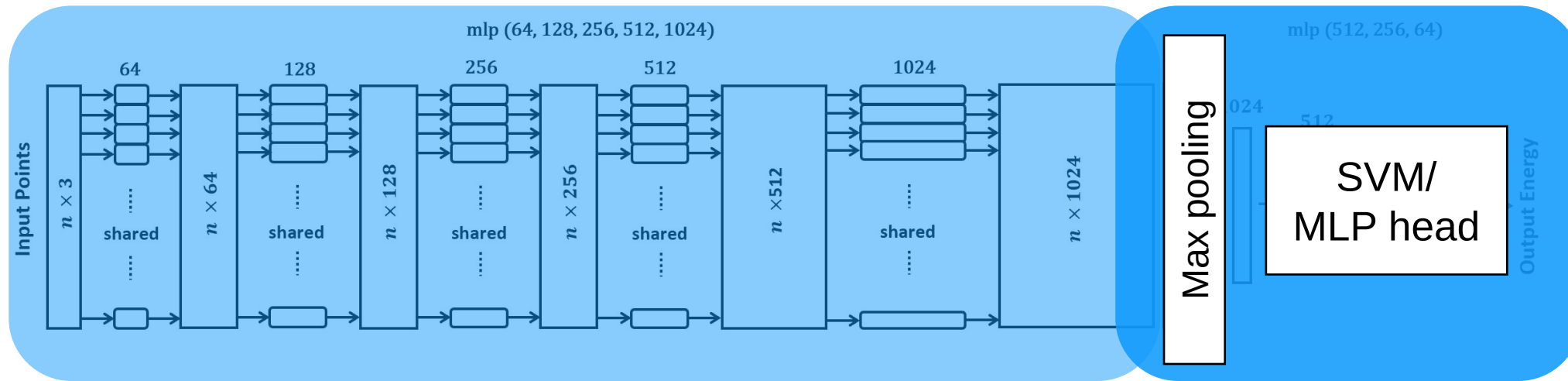
Table



Chair



Representation Learning



Unsupervised Learning
EBM Generative Feature Learning

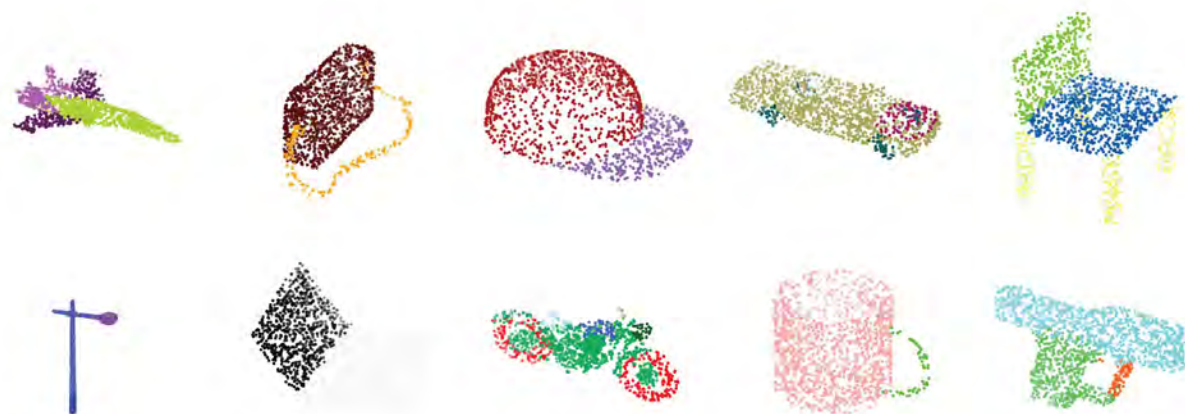
Supervised Learning
Downstream Task Learning

Representation Learning

Supervised Learning
Downstream Task Learning

Method	Accuracy
SPH [18]	79.8%
LFD [4]	79.9%
PANORAMA-NN [33]	91.1%
VConv-DAE [34]	80.5%
3D-GAN [38]	91.0%
3D-WINN [16]	91.9%
3D-DescriptorNet [44]	92.4%
Primitive GAN [19]	92.2%
FoldingNet [51]	94.4%
I-GAN [1]	95.4%
PointFlow [50]	93.7%
Ours	93.7%

Classification



Segmentation

1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

2. Representing

Energy-based Implicit Function
for 3D shape representation

3. Controlling

Energy-based Continuous
Inverse Optimal Control

Current challenge?

Unordered point set is non-trivial to deal with;
No good generative model for point cloud.

Why EBM helps?

No assisting network needed;
Derived from PointNet

How to model and sample?

Short-run MCMC by Langevin Dynamic
Regarded as k-layer generator

One more thing...

Representation learning
on Classification and Segmentation

1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta} \quad \text{on}$$

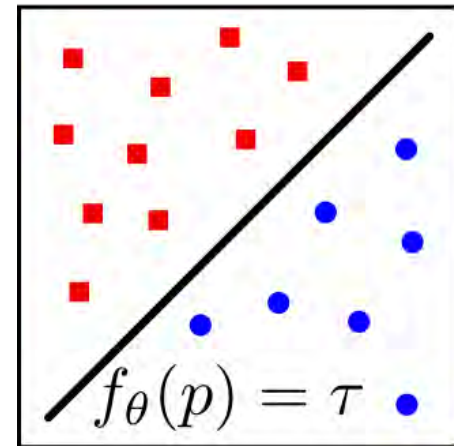
Energy-Based Model

2. Representing

Energy-based Implicit Function for 3D shape representation

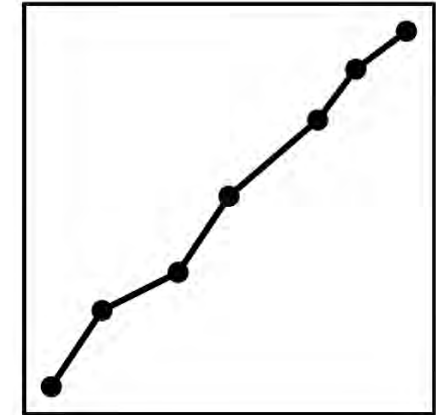
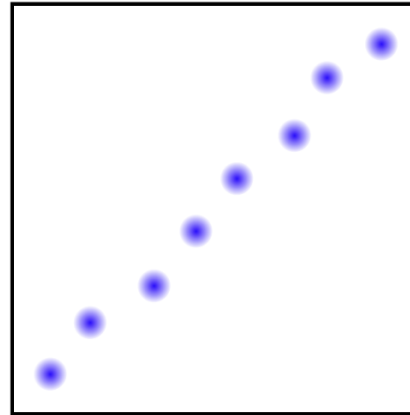
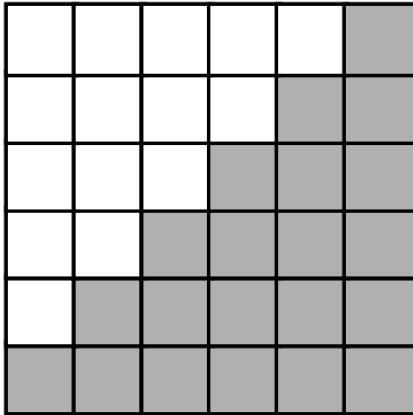
3. Controlling

Energy-based Continuous Inverse Optimal Control

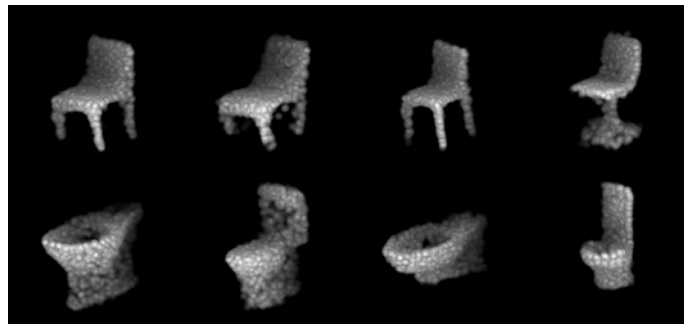


Implicit Representation

Represent a 3D shape



Voxel

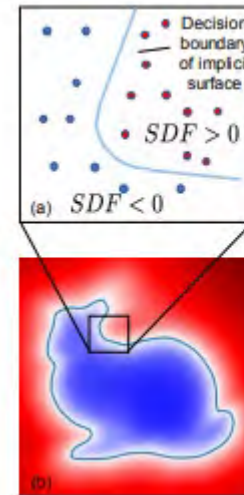
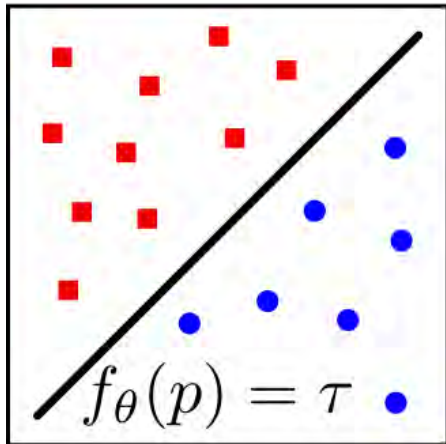


Point Cloud



Mesh

Related Work

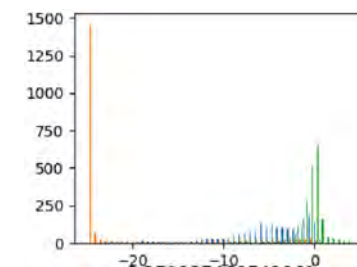
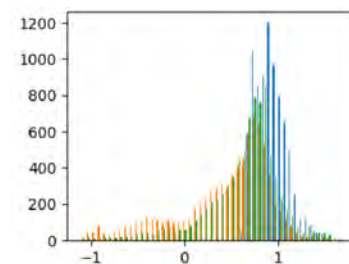
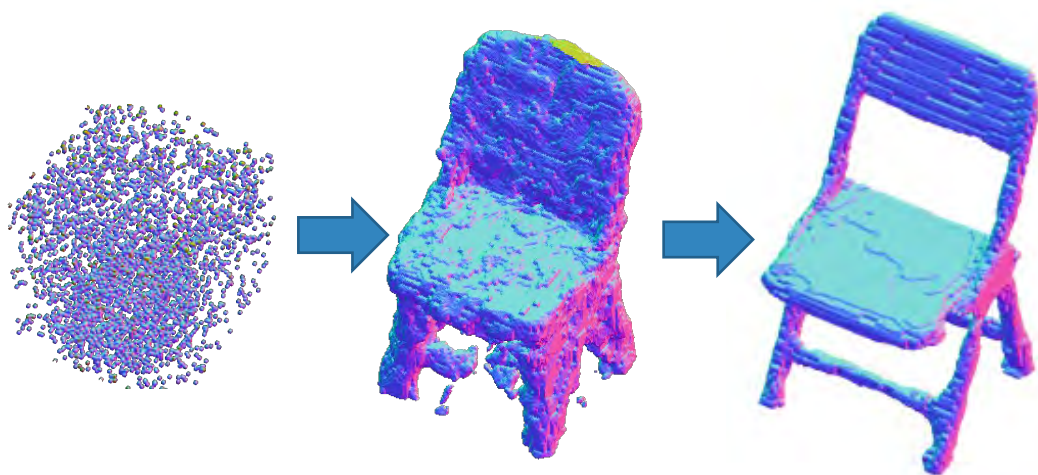


- **Occupancy Network (1 = on the surface; 0 = off the surface)**
 - Need sample negative points (point off the surface)
 - Train as a classifier.
- **Signed distance function (distance to the surface)**
 - Only work on watertight object.
 - Must have explicit definition of “in” and “out”
 - Need to calculate SDF over all training point.

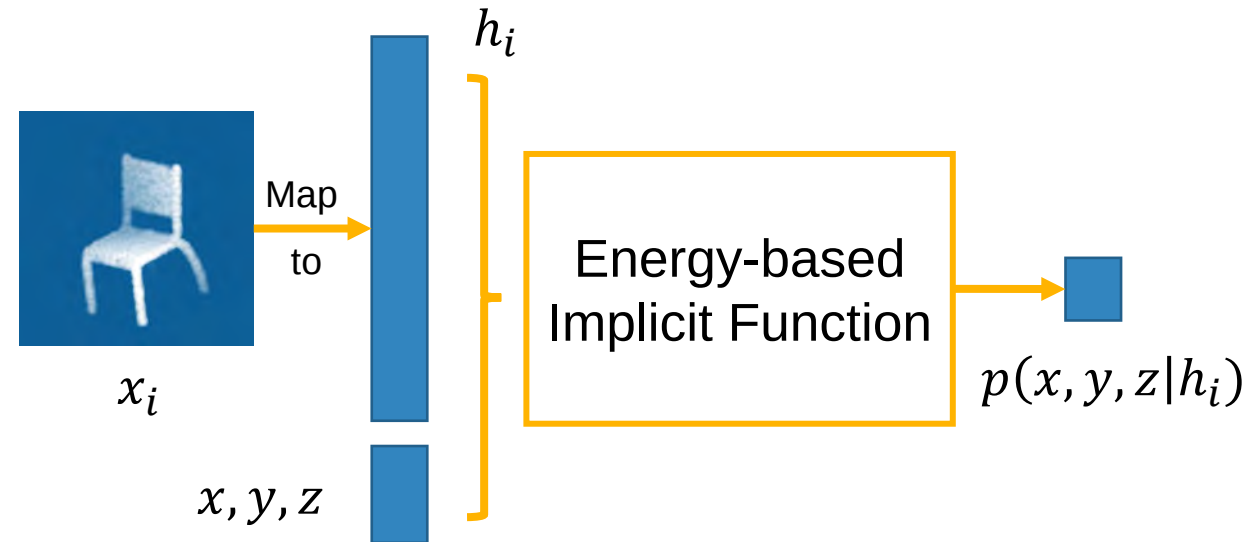
Energy-Based Implicit Function

- Defined on point

$$p_{\theta}(x, y, z) = \frac{1}{Z(\theta)} \exp f_{\theta}(x, y, z)$$



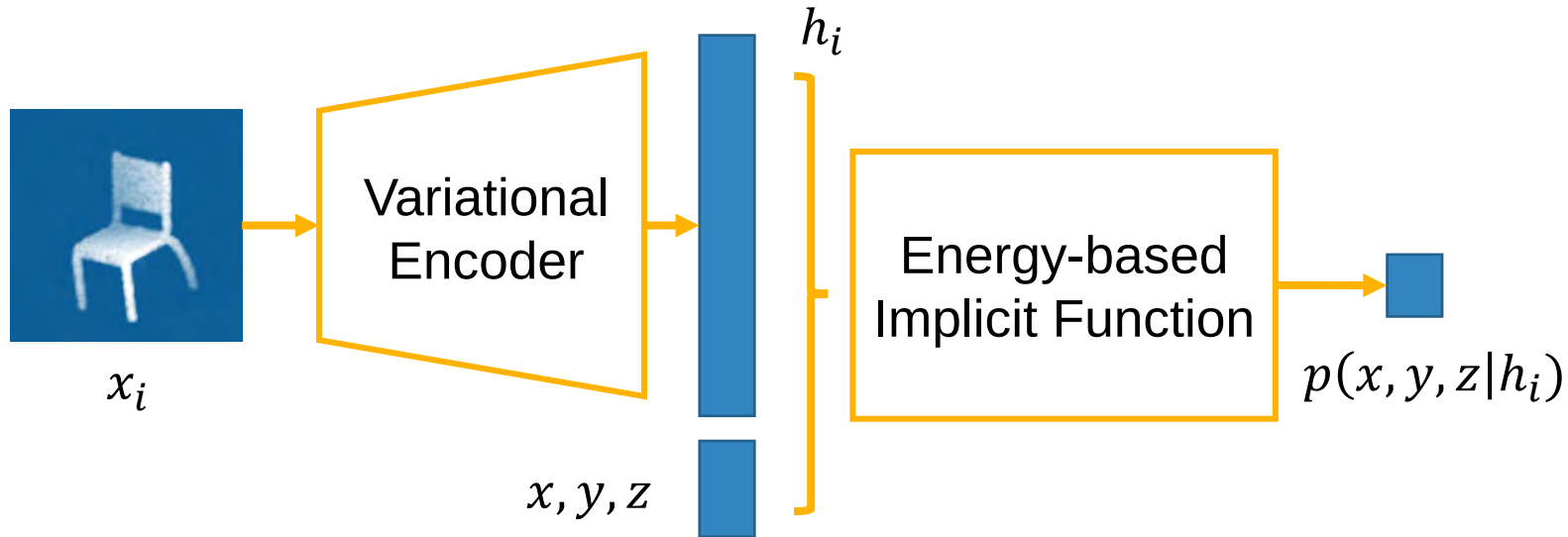
Conditional EBIF meets VAE



MLE Loss

Loss:
$$L(\phi | X_i) = E_{q_{\phi}(z | x_i)} \log p_{\theta}(x | z)$$

Conditional EBIF meets VAE



MLE Loss KL Loss

Variational Loss:
$$L(\phi|X_i) = E_{q_{\phi}(z|x_i)} \log p_{\theta}(x|z) - KL(q_{\phi}(z) \parallel p_0(z))$$

Reparametrized trick:
$$\sum_{\{x,y,z\} \in X_i} \log p_{\theta}(x, y, z | \mu_i + \epsilon \sigma_i) \quad \frac{1}{2} \sum_{j=1}^J (1 + 2 \log \sigma - \mu_j^2 - \sigma_j^2)$$

Importance Sampling

Maximum Likelihood Estimation use gradient descent:

$$\frac{\partial}{\partial \theta} l(\theta) = E_{q_{data}} \left[\frac{\partial}{\partial \theta} f_{\theta}(X) \right] - E_{p_{\theta}} \left[\frac{\partial}{\partial \theta} f_{\theta}(X) \right]$$

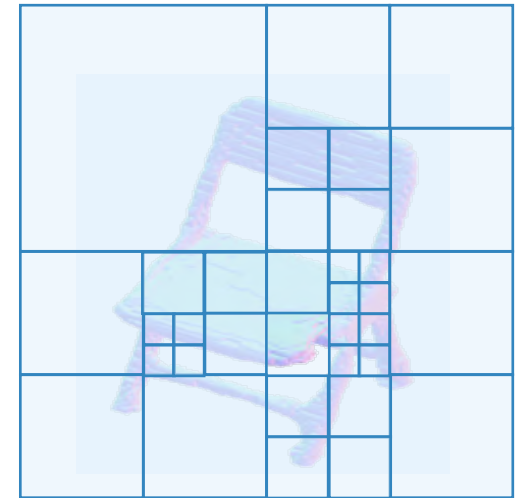
Importance Sampling

$$E_p[h(x)] = \int h(x)p(x)dx = \int \frac{p(x)}{q(x)} h(x)q(x)dx = E_q \left[\frac{p(x)}{q(x)} h(x) \right]$$

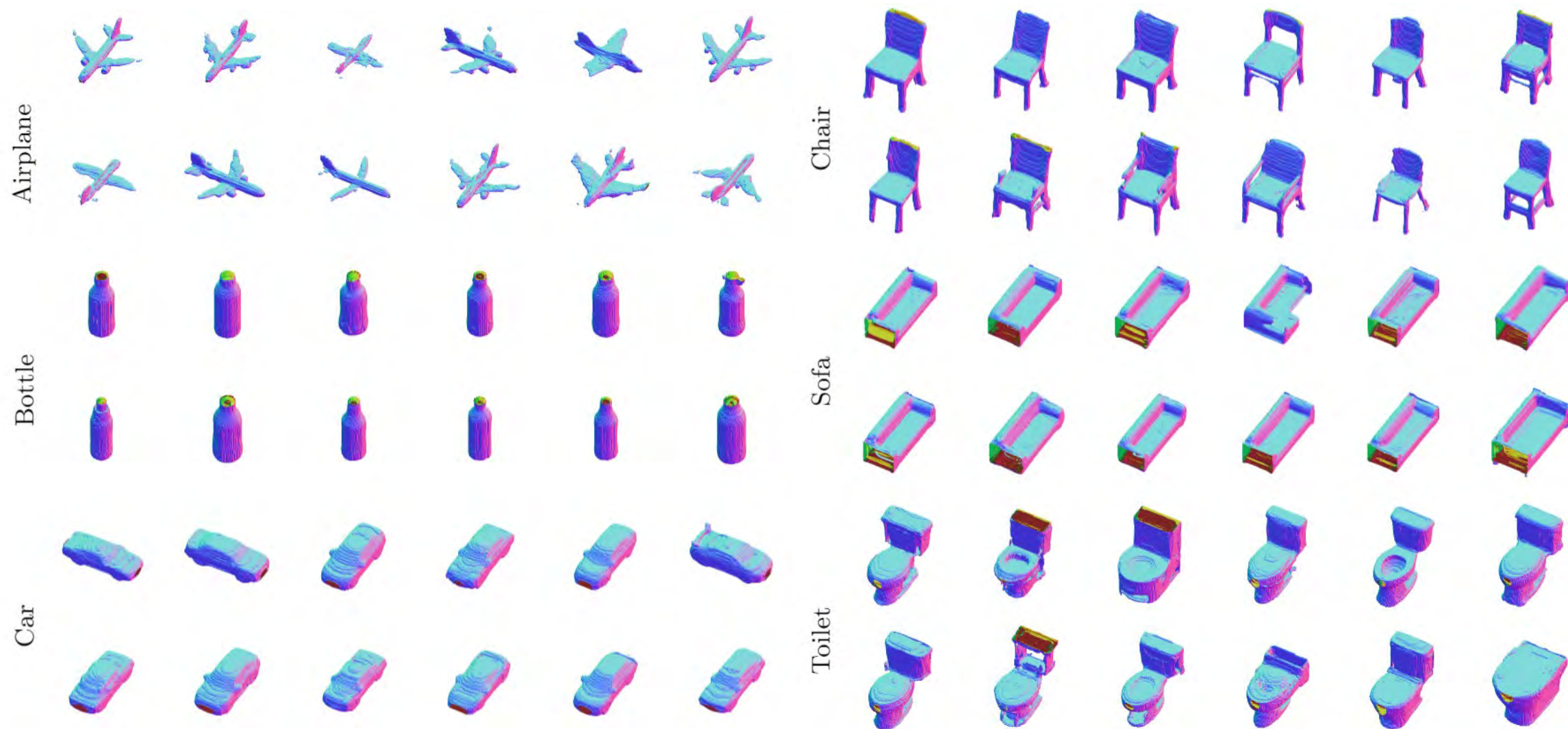
To make a better approximation:

1. The reference should be easy to sample. --- use uniform distribution
2. The reference should be not too far away from the target distribution --- Piecewise Uniform

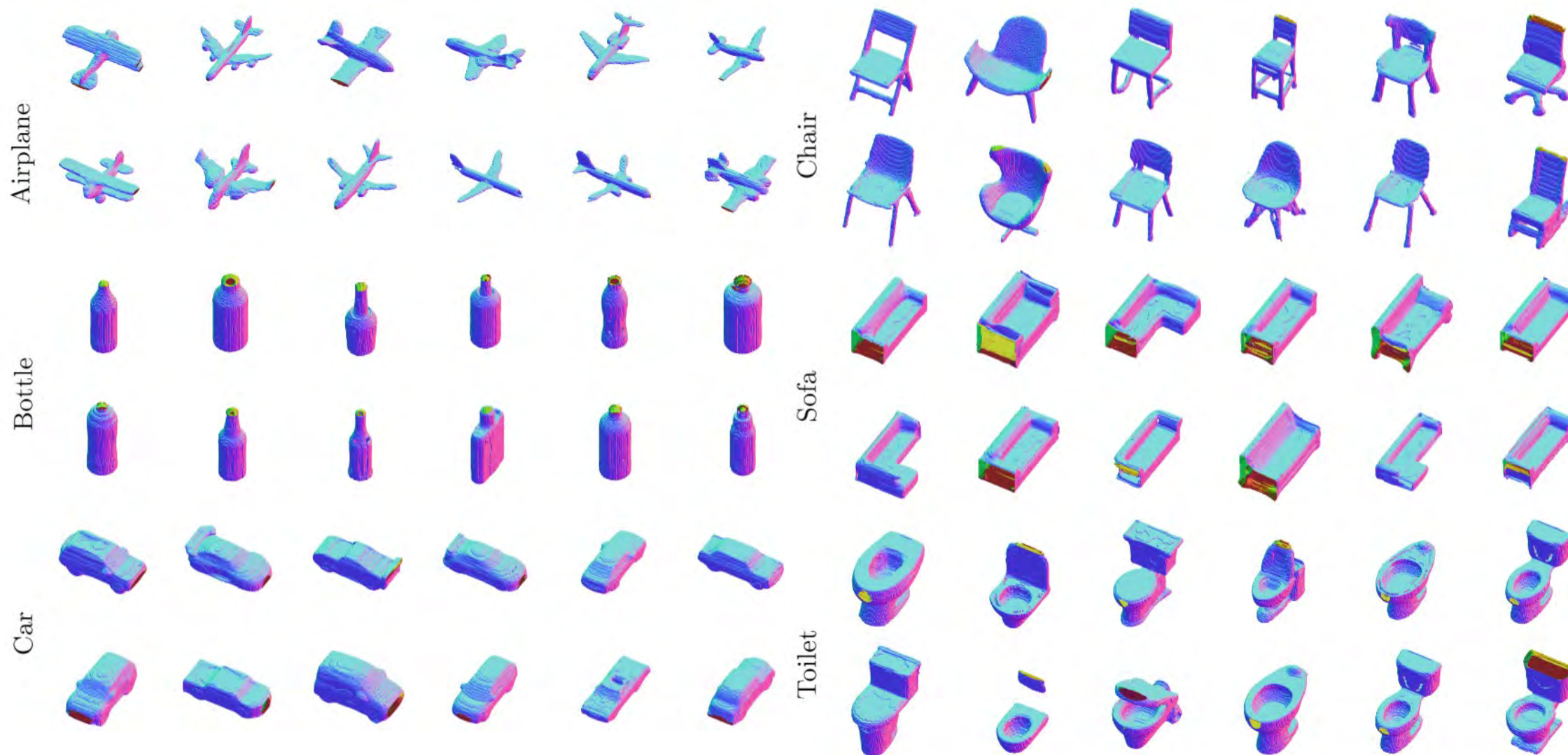
For point x in a specific cube G : $q(x) = \frac{1}{Z_q} \exp f_{\theta}(\bar{x})$, Where $\bar{x}$ is the center point of the cube G .



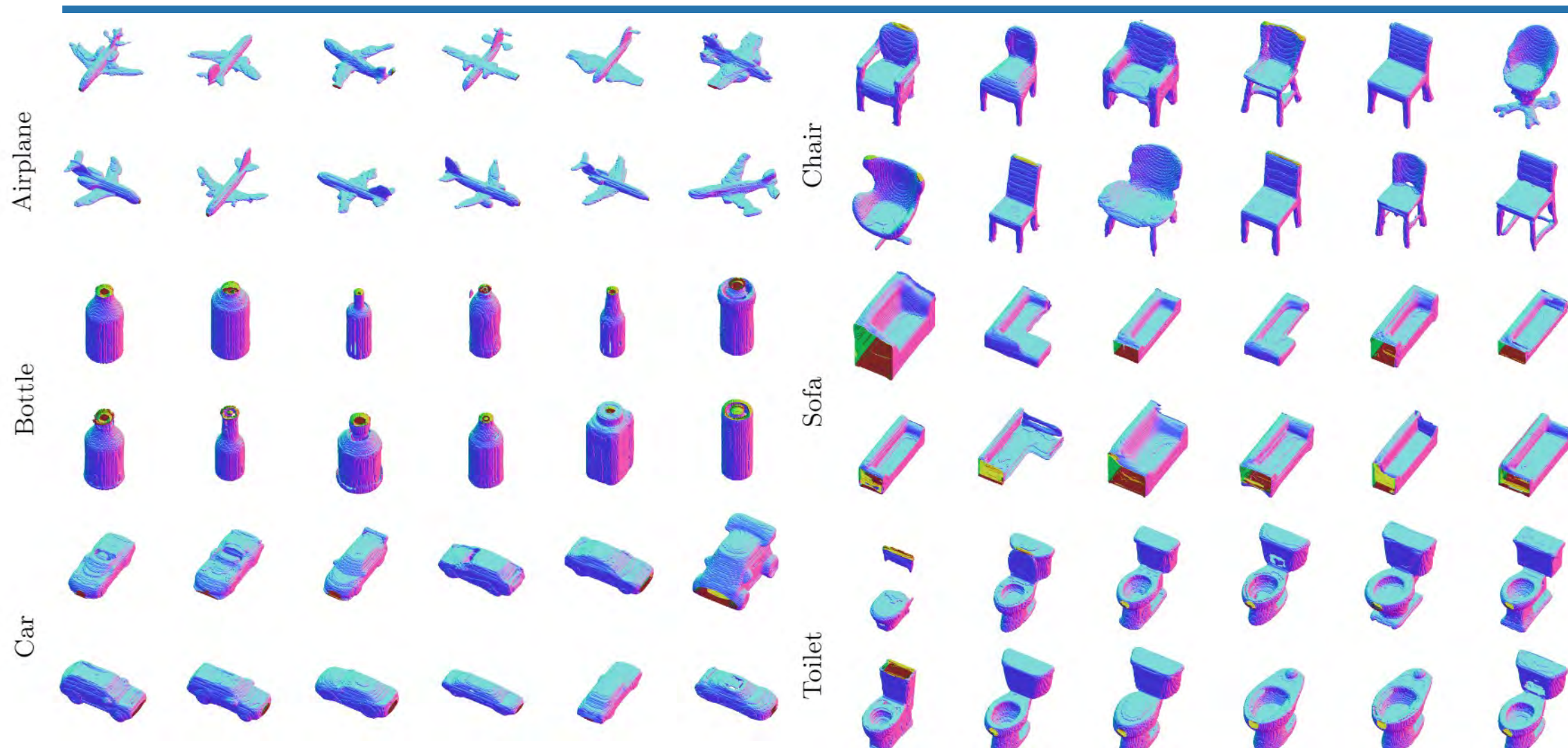
Generation Results



Reconstruction Results

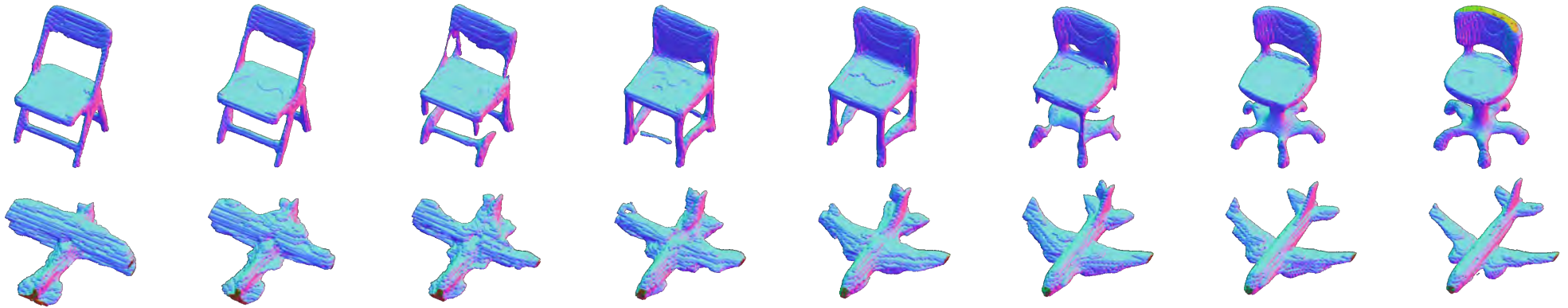


Testing reconstruction

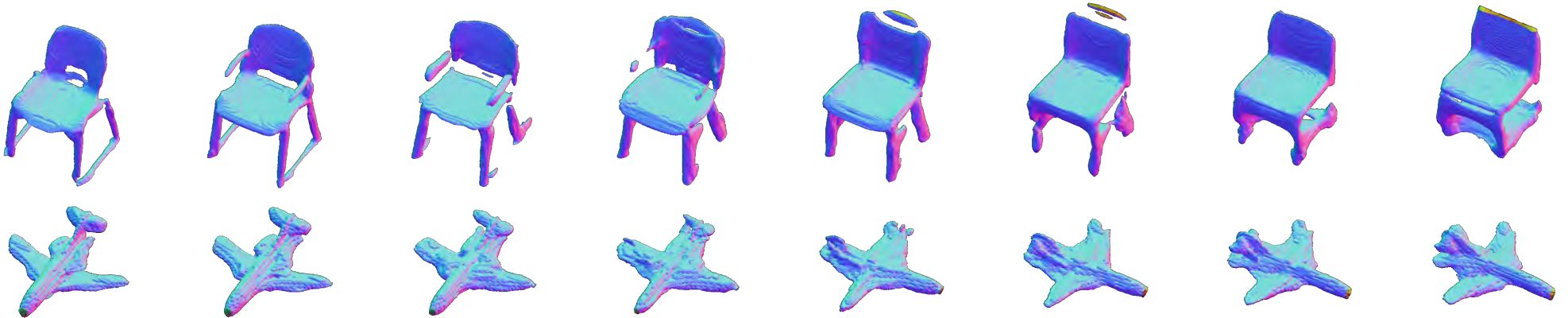


Interpolation Results

Training



Testing



1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

2. Representing

Energy-based Implicit Function for 3D shape representation

3. Controlling

Energy-based Continuous Inverse Optimal Control

Current challenge?

Cannot deal with non-watertight object;
Need human-defined function.

Why EBM helps?

A natural representation --- $p(x, y, z)$
No need to sample negative points

How to model and sample?

Importance Sampling
Multi-grid / volume-adaptive piecewise uniform

One more thing...

Cooperate with VAE
Good generation results

1. Learning

Generative PointNet: Energy-Based Learning on Unordered Point Sets

$$p_{\theta} = \frac{1}{Z_{\theta}} \exp f_{\theta} \quad \text{on}$$

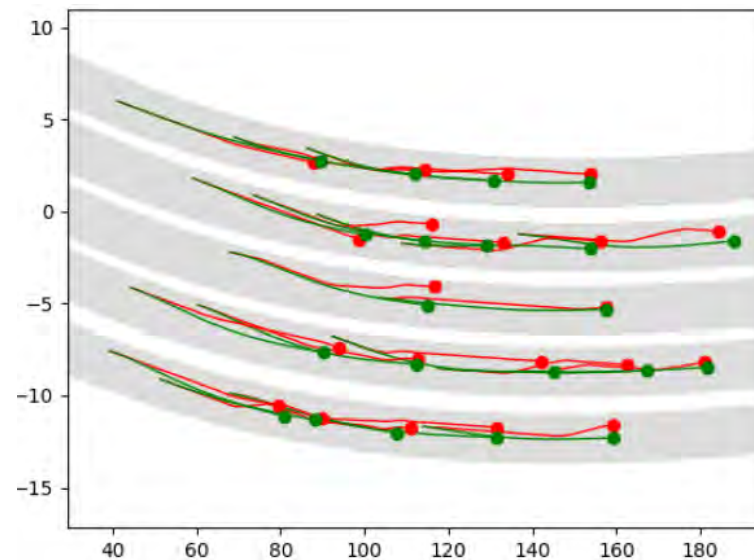
Energy-Based Model

2. Representing

Energy-based Implicit Function for 3D shape representation

3. Controlling

Energy-based Continuous Inverse Optimal Control



Inverse Optimal Control

Continuous Inverse Optimal Control

The MDP formulation for self-driving

State: $x = \{\text{longitude, latitude, speed, heading angle, acceleration, steering angle}\};$

x_t : State

change of
acceleration, change
of steering angle

u_t : Control

Dynamic : The
transition function.
 f is bicycle model.
$$x_{t+1} = f(x_t, u_t)$$

f : Dynamic Function

Continuous Inverse Optimal Control

The MDP formulation for self-driving

x_t : State

u_t : Control

f : Dynamic Function

C_θ : Cost Function

$$U = \arg \min_U C_\theta(X, U)$$

Continuous Inverse Optimal Control

The MDP formulation for self-driving

x_t : State

u_t : Control

f : Dynamic Function

C_θ : Cost Function

Learn from

Expert
Demonstration:
 $\{\tau_i\}$

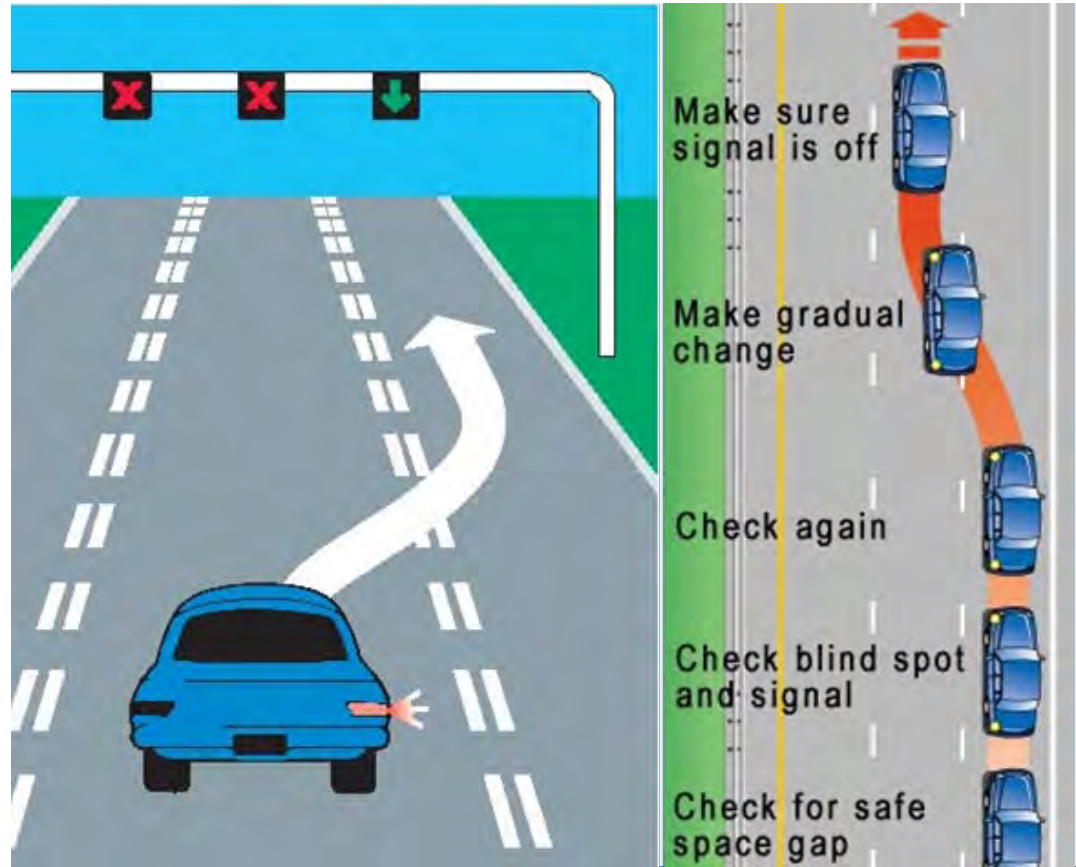
A sequence of state and control pair (X, U)

$$\theta = \arg \min_{\theta} C_{\theta}(\tau_i)$$

How human drive?

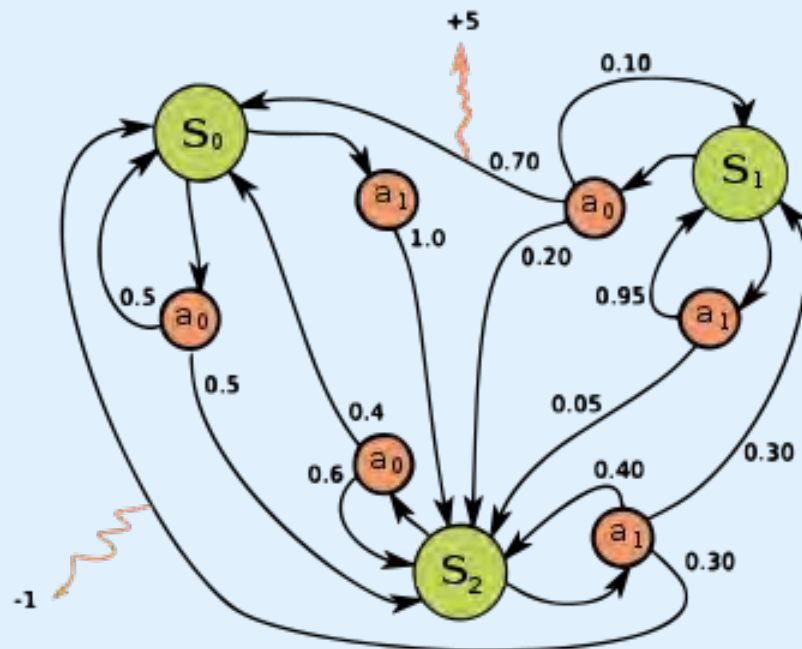


Instinct



Computation

How an agent drives?



Fast thinking: Policy Method

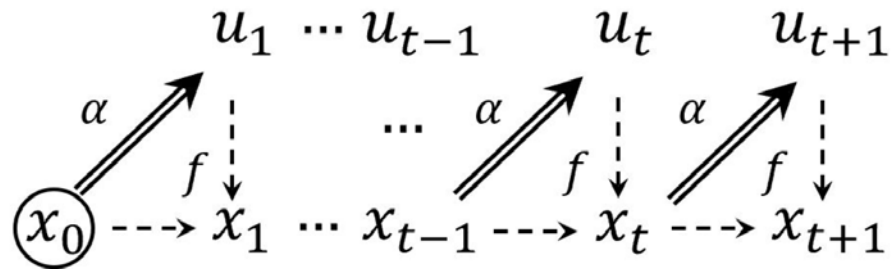


Slow thinking: Optimize Method

Two type of model for optimal control

$$u = \arg \min_u Q_\theta(x, u)$$

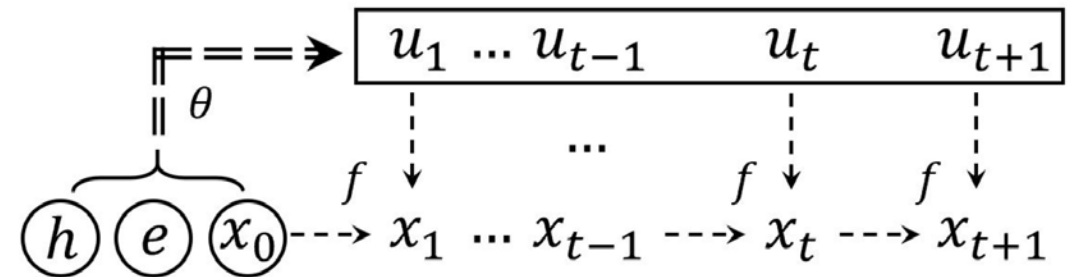
Q_θ : The expected future cost



Fast thinking: Policy Method

$$U = \arg \min_U C_\theta(X, U)$$

U : A sequence of control u C_θ : A sequence of cost c_θ



Slow thinking: Optimize Method

Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Conditional Energy-based Model:

$$p_{\theta}(\boldsymbol{\tau}|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

Where $Z_{\theta}(e, h)$ is the normalizing constant. $e :=$ environment; $h :=$ history.

- Previous work: Use Laplace approximation to approximate Z_{θ}

$$P(\mathbf{u}|\mathbf{x}_0) \approx e^{r(\mathbf{u})} \left[\int e^{r(\mathbf{u}) + (\tilde{\mathbf{u}} - \mathbf{u})^T \mathbf{g} + \frac{1}{2} (\tilde{\mathbf{u}} - \mathbf{u})^T \mathbf{H} (\tilde{\mathbf{u}} - \mathbf{u})} d\tilde{\mathbf{u}} \right]^{-1} = \left[\int e^{-\frac{1}{2} \mathbf{g}^T \mathbf{H}^{-1} \mathbf{g} + \frac{1}{2} (\mathbf{H}(\tilde{\mathbf{u}} - \mathbf{u}) + \mathbf{g})^T \mathbf{H}^{-1} (\mathbf{H}(\tilde{\mathbf{u}} - \mathbf{u}) + \mathbf{g})} d\tilde{\mathbf{u}} \right]^{-1} = e^{\frac{1}{2} \mathbf{g}^T \mathbf{H}^{-1} \mathbf{g}} |\mathbf{H}|^{\frac{1}{2}} (2\pi)^{-\frac{d_{\mathbf{u}}}{2}},$$

Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

$$p_{\theta}(\boldsymbol{\tau}|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

- Our method: Sampling-based Approach / Optimization-based Approach:

$$\frac{\partial}{\partial \theta} l(\theta) = \frac{1}{n} \sum_{i=1}^n \left[\frac{\partial}{\partial \theta} C_{\theta}(\tilde{\mathbf{x}}_i, \tilde{\mathbf{u}}_i, e, h) - \frac{\partial}{\partial \theta} C_{\theta}(\mathbf{x}_i, \mathbf{u}_i, e, h) \right]$$

Sampled through Langevin dynamics

or

Predicted through optimization method

Conditional EBM

Fast thinking: Policy Method

Slow thinking: Optimize Method

$$p_{\theta}(\boldsymbol{\tau}|e, h) = p_{\theta}(\mathbf{u}|e, h) = \frac{1}{Z_{\theta}(e, h)} \exp[-C_{\theta}(\mathbf{x}, \mathbf{u}, e, h)]$$

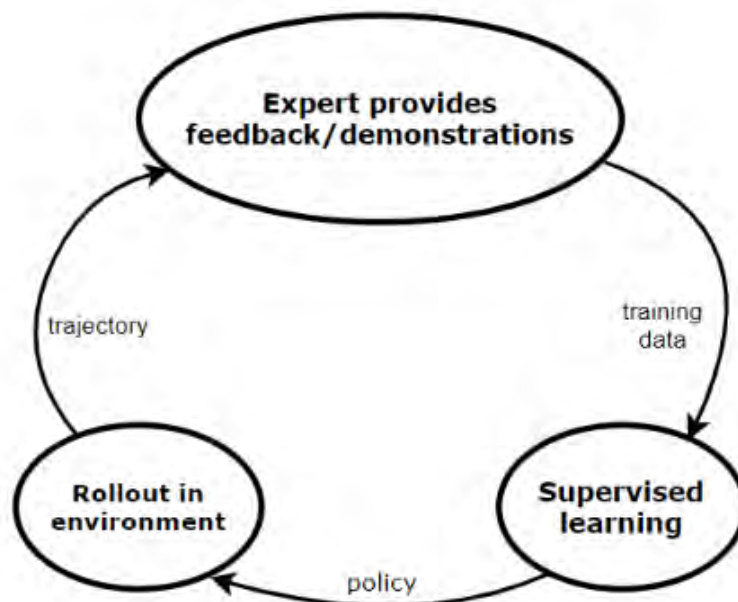
- Sample-based approach:
 - More exploration
 - Finding corner case
 - Better generalization
- Support more complex cost function:
 - 1D CNN
 - LSTM
 - MLP

Two type of model for optimal control

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Previous work: Imitate policy learnt from expert demonstration. Result used directly.



Two type of model for optimal control

Fast thinking: Policy Method

Slow thinking: Optimize Method

- Our method: Imitate policy learn from the optimized result (slow thinking result)
- A generator is used as a fast initializer of the sampling (or the optimization):

$$q_{\alpha}(u, \xi | e, h) = q_{\alpha}(u | \xi, e, h) p(\xi)$$

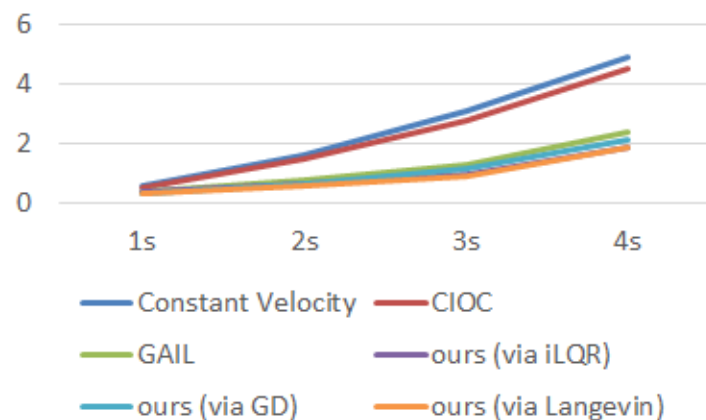
- Loss for encoder:

$$L_g(\alpha) = \frac{1}{n} \sum_{i=1}^n \| \tilde{u}_i - q_{\alpha}(\xi, e, h) \|^2$$

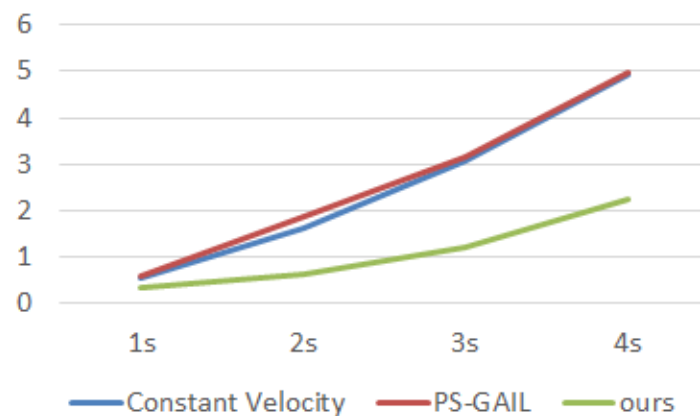
- Result is used as the initialization of the sampling / optimization

EBIOC: Predicted trajectories

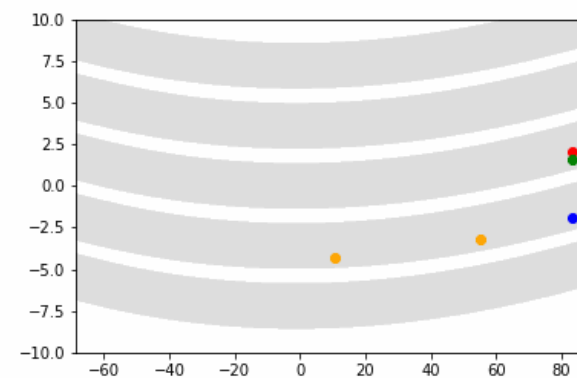
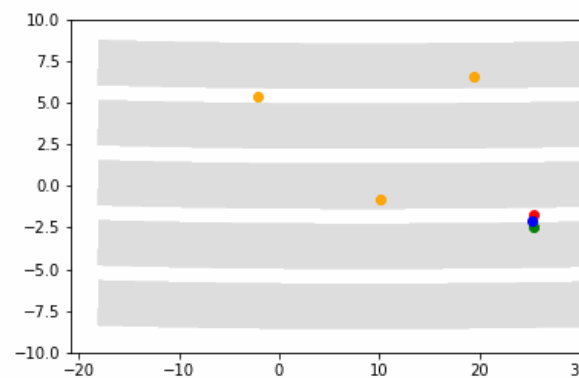
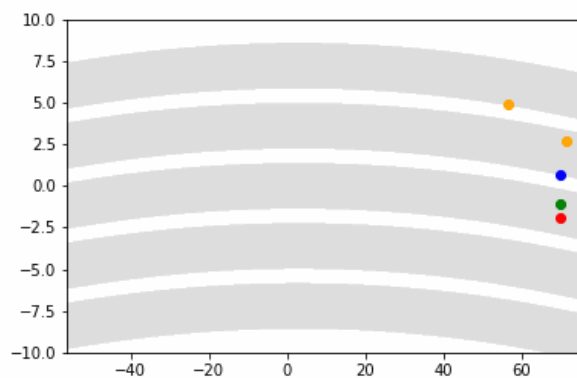
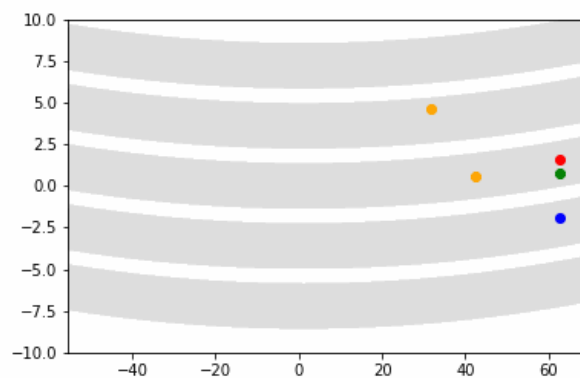
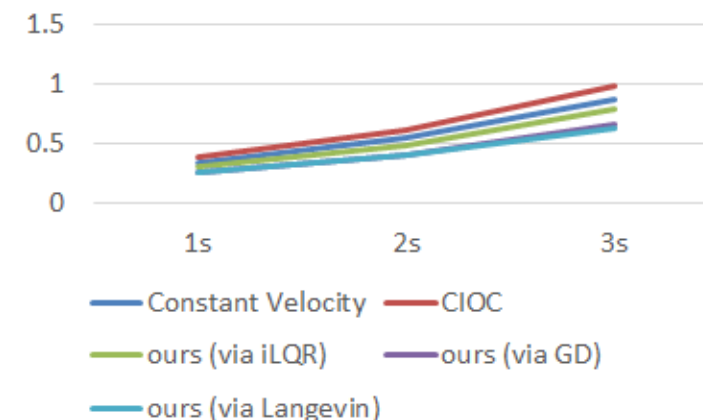
NGSIM single-agent



NGSIM multi-agent

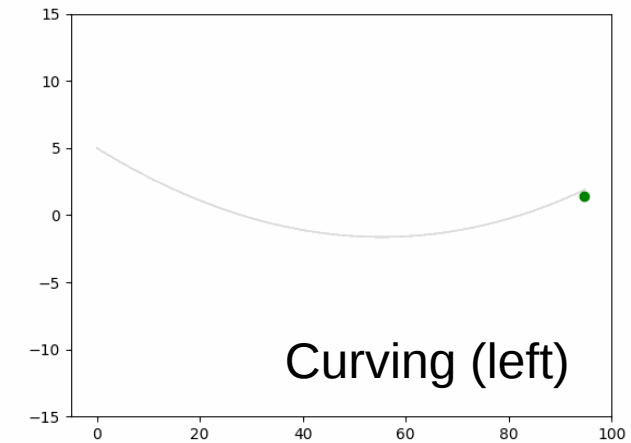
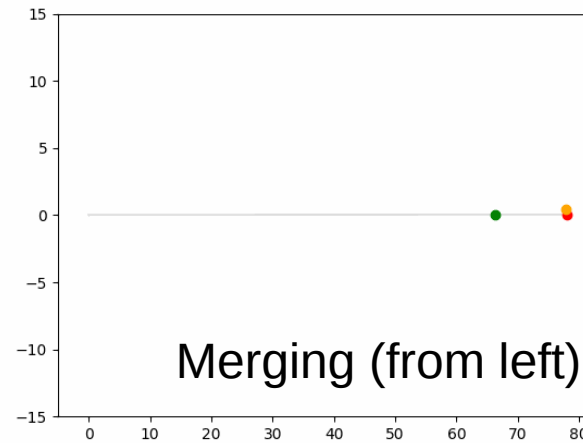
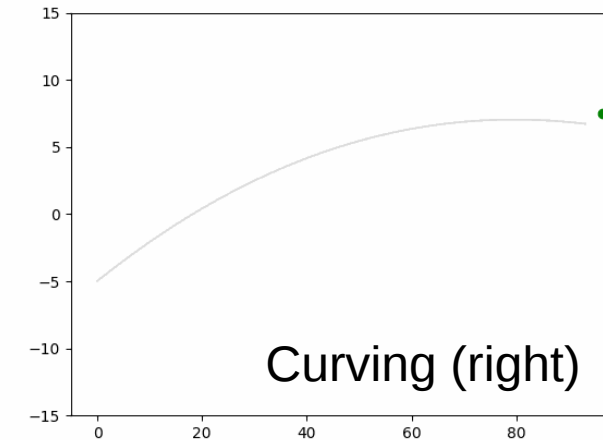
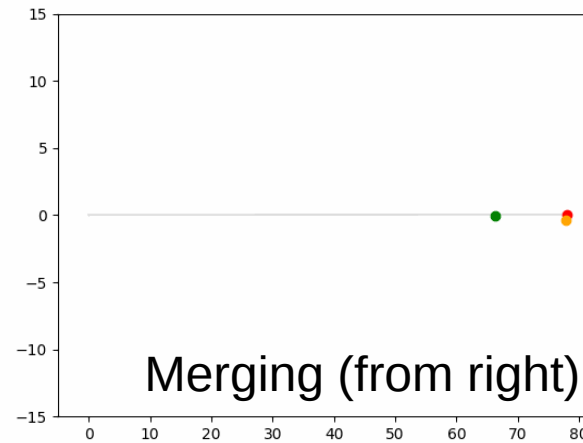
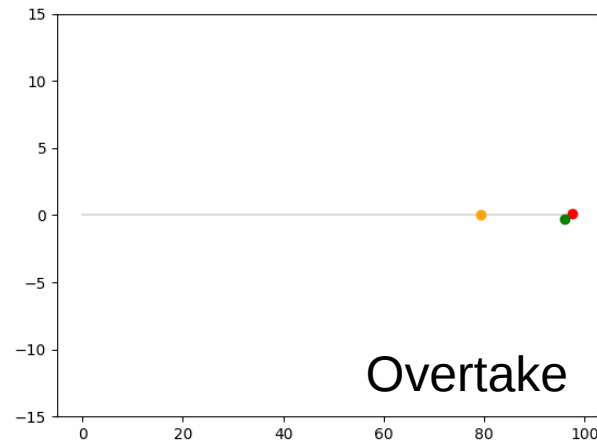


ISEE single-agent



■ Ground Truth; ■ Ours; ■ GAIL; ■ Other Vehicle; ■ Lane.

EBIOC: Toy examples

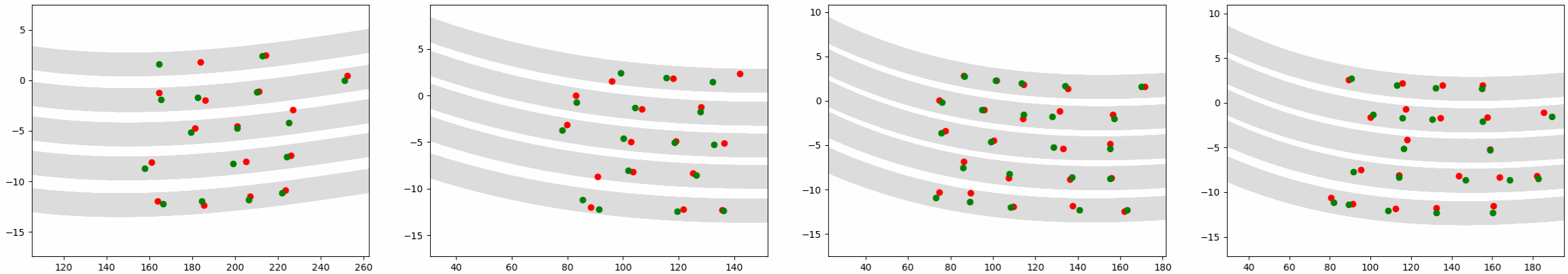


■ Green: predicted trajectories; ■ Orange: other vehicles; ■ Red: Rollout if keep constant control.

EBIOC: Multi-agent Prediction Result

- Meta-cost: sum of each cost = Product of probability = A joint probability distribution
- Assumption: Fully cooperate + Share information

$$p_{\theta}(\mathbf{U}|\mathbf{e}, \mathbf{h}) = \prod_{k=1}^K p_{\theta}(\mathbf{u}^k|\mathbf{e}, h^k) = \frac{1}{Z_{\theta}(\mathbf{e}, \mathbf{h})} \exp \left[- \sum_{k=1}^K c_{\theta}(\mathbf{x}^k, \mathbf{u}^k, \mathbf{e}, h^k) \right]$$



Multi-agent prediction on NGSIM US101 dataset (■ Lane; ■ Ground Truth; ■ Ours)

1. Learning

Generative PointNet: Energy-Based
Learning on Unordered Point Sets

2. Representing

Energy-based Implicit Function
for 3D shape representation

3. Controlling

Energy-based Continuous
Inverse Optimal Control

Current challenge?

Trade off between exploration and exploitation
Trade off between efficiency and accuracy

Why EBM helps?

Sample-based: Better exploration
Combine policy method and optimization

How to model and sample?

Sample-base: Langevin Dynamic
Optimized-based: iLQR

One more thing...

Multi-agent setting
Joint EBM cooperative learning

Publications

- 2022.03 Jianwen Xie, Yaxuan Zhu, **Yifei Xu**, Dingcheng Li, Ping Li " [Generative Learning with Latent Space Flow-based Prior Model](#)" *In review*
- 2022.02 **Yifei Xu**[†], Jingqiao Zhang[†], Ru He[†], Liangzhu Ge[†], Chao Yang, Cheng Yang, Ying Nian Wu " [SAS: Self-Augmented Strategy for Language Model Pre-training](#)" *In Proc. 36th AAAI Conference on Artificial Intelligence (AAAI) 2022*
- 2021.06 **Yifei Xu**[†], Jianwen Xie[†], Zilong Zheng, Song-Chun Zhu, Ying Nian Wu " [Generative PointNet : Deep Energy-Based Learning on Point Sets for 3D Generation and Reconstruction](#)" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2021.*
- 2020.02 **Yifei Xu**, Jianwen Xie, Tianyang Zhao, Chris Baker, Yibiao Zhao, Ying Nian Wu " [Energe-based Continous Inverse Optimal Control](#)" *Journal version submitted to TNNLS; NeurIPS workshop on Machine Learning for Autonomous Driving, 2020*
- 2018.11 Tianyang Zhao, **Yifei Xu**, Mathew Monfort, Wongun Choi, Chris Baker, Yibiao Zhao, Yizhou Wang, Ying Nian Wu " [Convolutional Spatial Fusion for Multi-Agent Trajectory Prediction](#)" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2019.*
- 2017.03 Jianwen Xie, **Yifei Xu**, Erik Nijkamp, Ying Nian Wu, Song-Chun Zhu " [Generative Hierarchical Structure Learning of Sparse FRAME Models](#)" *IEEE Conference on Computer Vision and Pattern Recognition (CVPR) 2017.*

Reference

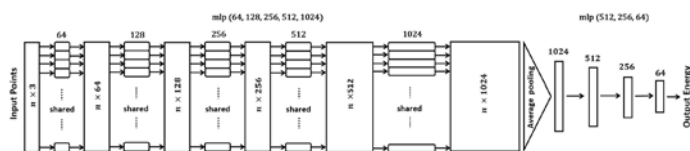
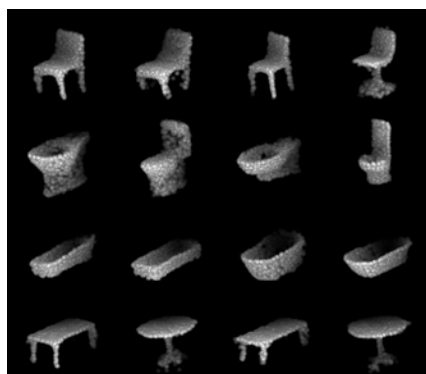
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Thanks

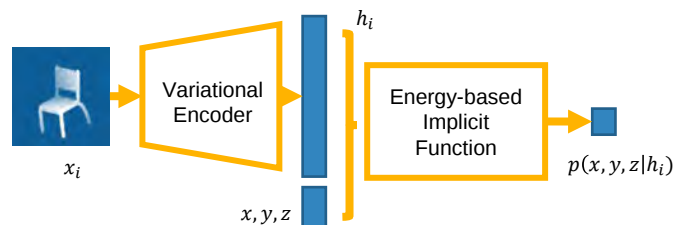


Q&A

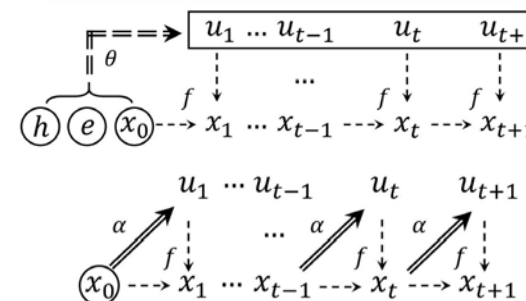
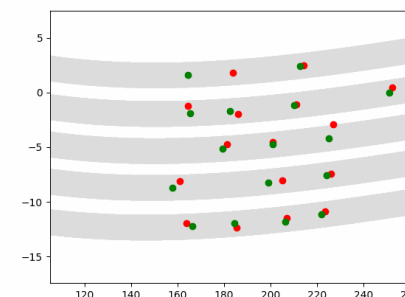
$$\text{Energy-based Model: } p(X) = \frac{1}{Z} \exp f(X)$$



Point Cloud



Implicit Function



Control